Quadratic Formula

Theorem Suppose a, b, and c are any real numbers, with the exception that $a \neq 0$. The quadratic equation:

$$a\,x^2 + b\,x + c = 0$$

has the two solutions:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example Solve the following quadratic equation:

$$x^2 - 4\,x - 7 = 0$$

Here $a = 1$, $b = -4$ and $c = -7$. The quadratic formula says that the two solutions to this equation are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-7)}}{2(1)}$$

$$= \frac{4 \pm \sqrt{16 + 28}}{2} = \frac{4}{2} \pm \frac{\sqrt{44}}{2} = \frac{4}{2} \pm \frac{\sqrt{4 \cdot 11}}{2}$$

$$= 2 \pm \frac{\sqrt{4}\sqrt{11}}{2} = 2 \pm \frac{2\sqrt{11}}{2} = 2 \pm \sqrt{11}$$

so that $x = 2 \pm \sqrt{11}$ satisfies the given equation.

Mnemonic Device

Here's a story that makes it easy to remember your quadratic formula:

> Once there was a bad boy $(-b)$,
> who was kind of wishy-washy $(\pm)$
> about attending a radical party $\sqrt{\ }$
> because the boy was kind of square (b^2).
> When he arrived he was kind of nervous, or negative (-)
> about meeting these four awesome chicks $(4ac)$
> it was ALL OVER at 2 antemeridian $(2a)$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$